LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

SIXTH SEMESTER – **APRIL 2023**

16/17/18UMT6MC04 – GRAPH THEORY

Date: 08-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

PART - A

Answer ALL questions. **(10 x 2 = 20 marks)**

1. Construct a graph with five vertices each having degree 2.
2. Define connected graph.
3. Draw a complete graph on 6 vertices.
4. Define a Hamiltonian graph and give an example.
5. Prove that there is one and only one path between every pair of vertices in a tree.
6. Define eccentricity of a vertex.
7. Define a cut-set in a connected graph.
8. Define a separable graph and give an example.
9. Define a planar graph.
10. What is meant by digraph?

PART - B

Answer any FIVE questions. **(5 x 8 = 40 marks)**

11. Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two nonempty disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in V_1 and the other end in V_2.
12. Illustrate union and intersection of graphs with suitable examples.
13. Prove that every tree has either one or two centres.
14. Prove that the vertex connectivity of any graph G can never exceed the edge connectivity of G.
15. Show that a graph of n vertices is a complete graph if and only if its chromatic polynomial is given by $P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \dots (\lambda - n + 1)$.
16. Prove that a connected graph G is Euler graph if and only if it can be decomposed into circuits.
17. Explain any two applications of the spanning tree in real life.
18. Prove that the complete graph of five vertices is nonplanar.

PART - C

Answer any TWO questions. **(2 x 20 = 40 marks)**

19. a) Show that a simple graph with n vertices and k components can have at most $\frac{(n-k)(n-k+1)}{2}$ edges.
    b) Prove that a connected graph G is Euler graph if and only if all vertices of G are of even degree.
20. a) If n is odd number, prove that there are $(n-1)/2$ edge-disjoint Hamiltonian circuits in a complete graph with n vertices.
    b) Prove that a tree with n vertices has $(n-1)$ edges.

21. a) Prove that every circuit has an even number of edges in common with any cut-set.

    b) Prove that the ring sum of any two cut-sets in a graph is either a third cut-set or an edge disjoint union of cut-sets.

22. a) Prove that a connected planar graph with n vertices and e edges has $e - n + 2$ regions.

    b) Show that a graph with at least one edge is 2-chromatic if and only if it has no circuits of odd length.

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